On the warped heterotic axion

Keshav Dasgupta, Hassan Firouzjahi, Rhiannon Gwyn

Ernest Rutherford Physics Building, McGill University,
3600 University Street, Montréal QC, H3A 2T8 Canada
E-mail: keshav@hep.physics.mcgill.ca, firouz@hep.physics.mcgill.ca,
gwynr@hep.physics.mcgill.ca

ABSTRACT: We study the axion in a warped heterotic background. It is shown that the axion decay constant, f_a, is sensitive to the warped mass scale of the throat. As an explicit model, we construct a novel AdS-like geometry in heterotic string theory. We demonstrate that in this background f_a is given by the mass scale of the longest throat in the compactification. The question of obtaining f_a within the required bound $10^9 - 10^{12}$ GeV is reduced to the construction of a throat inside the heterotic string theory compactification with warped mass scale in the above range. This provides a natural mechanism for realising the axion in heterotic string theory.

Contents

1. Introduction

The axion was proposed as a plausible solution to the strong CP problem of QCD [1, 3, 4]. More specifically, the QCD action contains the non-perturbative term

$$S_\theta = \frac{\theta}{32\pi^2} \int d^4x \, \epsilon^{\alpha\beta\gamma\lambda} \mathbf{tr} \, F_{\alpha\beta} F_{\gamma\lambda} \,, \tag{1.1}$$

where $\mathbf{tr}$ represents the trace in the three-dimensional representation of SU(3) and θ is an angular parameter. Given (1.1) one might conclude that $|\theta|$ could take any value between 0 and 2π, but in fact there are strong observational constraints on $|\theta|$. A non-zero value of θ results in a nonvanishing dipole moment for the neutron. The recent upper bound on this dipole moment [5] implies that $|\theta| < 10^{-9}$. Explaining this small value is the strong CP problem.

The axion, a, is a massless scalar field with the global shift symmetry $a \to a + \epsilon$. This symmetry is primarily broken by QCD instanton effects with the coupling

$$\Delta S = \frac{a}{32\pi^2} \int d^4x \, \epsilon^{\alpha\beta\gamma\lambda} \mathbf{tr} \, F_{\alpha\beta} F_{\gamma\lambda} \,. \tag{1.2}$$

Combined with its kinetic energy, the action for the axion field is

$$S_a = \int d^4x \left(-\frac{f_a^2}{2} \partial_\mu a \partial^\mu a + \frac{a}{32\pi^2} \epsilon^{\alpha\beta\gamma\lambda} \, \mathbf{tr}\, F_{\alpha\beta} F_{\gamma\lambda} \right)$$

$$S_{\tilde{a}} = \int d^4x \left(-\frac{1}{2} \partial_\mu \tilde{a} \partial^\mu \tilde{a} + \frac{\tilde{a}}{32\pi^2 f_a} \epsilon^{\alpha\beta\gamma\lambda} \, \mathbf{tr}\, F_{\alpha\beta} F_{\gamma\lambda} \right) . \qquad (1.3)$$

In the second line above the rescaled axion field $\tilde{a}$ is introduced such that its kinetic energy has the standard form. The parameter f_a is called the axion decay constant and is inversely related to the axion mass.

Given the similarity of (1.1) and (1.2), one can infer that the physical quantities are independent of θ. The presence of the term (1.1) can be absorbed by a constant shift in axion field via $a \to a - \theta$. Alternatively, one can think of the effect of adding the axion field into the model as promoting θ to a dynamical field which is naturally relaxed to zero by its vacuum expectation value.

There are strong astrophysical and cosmological bounds on the axion decay constant f_a:

$$10^9 \,\mathrm{GeV} < f_a < 10^{12} \,\mathrm{GeV} . \qquad (1.4)$$

If f_a is less than $10^9 \,\mathrm{GeV}$, the axion coupling will be too strong, leading to the production of too many axions. This would accelerate the evolution of stars such as red giants, by transporting their energies into the outer regions more efficiently and shortening their lifetimes. On the other hand, if f_a is more than $10^{12} \,\mathrm{GeV}$, the axion coupling will be too weak, leading to the production of too much axionic dark matter in the universe. For more details on the cosmological bounds and related issues to do with axions see the review papers [6 – 10] and references therein.

In this paper we would like to construct axions in models of warped heterotic string theory. The paper is organized as follows. In section 2 we present the construction of the axion in warped heterotic theory, without specifying the background. In section 3 we focus on building a new AdS-like warped geometry in heterotic string theory. It is shown that in this background f_a can be lowered to values within the phenomenological window. In section 4 we present another warped geometry in heterotic string theory where f_a may fall within the required window. A brief conclusion and discussions are presented in section 5.

2. Warped heterotic axions

Axions can naturally be embedded in string theory; for a review see [11 – 14] and references therein. The problem with axion construction in conventional string theory models is that it typically results in an axion decay constant higher than the range of phenomenologically allowed values. This was extensively studied in [13] with the conclusion that for string scale m_s comparable to M_P the axion decay constant is typically of order $10^{16} \,\mathrm{GeV}$, too big to be allowed.

Here we consider whether it is possible to exploit the effects of warping to reduce the scale of f_a in heterotic string theory. Models of warped axions in the context of a five-dimensional Randall-Sundrum scenario were presented in [15 – 18] .

2.1 The effect of warping on f_a

We take heterotic string theory compactified on a six-dimensional manifold $\mathcal{M}$, which in general will not be a Calabi-Yau (CY) manifold. Furthermore, the background spacetime is a warped geometry where the metric is of the form

$$ds^2 = h_w^2 \eta_{\mu\nu} dx^\mu dx^\nu + g_{mn}(y) dy^m dy^n. \tag{2.1}$$

Here h_w is the warp factor and $y^m, y^n, \ldots$ represent the coordinates along the manifold $\mathcal{M}$. The warp factor depends on the internal coordinates. Here and below the small Latin indices $m, n, \ldots$ represent the coordinates tangential to $\mathcal{M}$, the Greek indices $\mu, \nu, \ldots$ represent the Minkowski coordinates and the capital indices $M, N, \ldots$ are ten-dimensional indices.

The axion arises from the zero mode of the NS field potential B_{MN} upon compactification. Depending on which component of B_{MN} one considers, there are two types of axion. If all components of B_{MN} are tangential to the Minkowski spacetime and are constant over $\mathcal{M}$, then the resulting axion is said to be "model-independent." $B_{\mu\nu}$ is Hodge dual in 4 dimensions to a scalar field which is our axion. Historically, the name comes from the belief that the details of the compactification would not influence the axion construction. We shall show that this is a misnomer in the context of warped compactification since here the axion depends sensitively on the details of the compactification. On the other hand, one can consider B_{mn} with components tangential to the compactified manifold $\mathcal{M}$. From the point of view of a four-dimensional observer this is a scalar which is then termed a "model-dependent" axion.

The ten-dimensional heterotic string theory action in the Einstein frame is

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left(R - \frac{1}{2} \partial_M \phi \partial^M \phi - \frac{e^{-\phi}}{2} H^2 - \frac{\alpha'}{120} e^{-\frac{\phi}{2}} \operatorname{Tr} F^2 \right) \tag{2.2}$$

where R is the Ricci scalar, ϕ the dilaton, F the $E_8 \times E_8$ or SO(32) gauge field with trace in the adjoint representation of the gauge group and H the NS-NS three-form constructed from the two-form potential B as well as the curvature and the gauge field F. Here κ is the ten-dimensional gravitational coupling given by $4\pi \kappa^2 = (2\pi\sqrt{\alpha'})^8 \equiv m_s^{-8}$, where m_s is defined as the string mass scale. Finally, Tr represents the trace in the adjoint representation and $\mathbf{tr} = \operatorname{Tr}/60$.

To take into account the effect of warp factors on the graviton and NS field zero modes, we consider dimensional reduction of each mode separately. The four-dimensional action for the zero mode of the graviton is calculated by perturbing the background in (2.1) such that $\eta_{\mu\nu} \to \bar{g}_{\mu\nu}(x^\alpha)$ where $\bar{g}_{\mu\nu}(x^\alpha)$ is the metric observed by the four-dimensional observer. The Ricci scalar is decomposed into $R = h_w^{-2} \bar{R} + \ldots$ where $\bar{R}$ represents the Ricci scalar constructed from $\bar{g}_{\mu\nu}(x^\alpha)$.

The four-dimensional action for the zero mode of the graviton, $S_g^{(0)}$, is given by

$$\begin{aligned} S_g^{(0)} &= \frac{1}{2\kappa^2} \int d^4x \sqrt{-\bar{g}}\, \bar{R} \int d^6y \sqrt{g_{(6)}}\, h_w^2(y) \\ &= \frac{M_P^2}{2} \int d^4x \sqrt{-\bar{g}}\, \bar{R}, \end{aligned} \tag{2.3}$$

where $g_{(6)}$ is the determinant of the internal metric and

$$M_P^2 = \frac{1}{\kappa^2} \int d^6y \, \sqrt{g_{(6)}} \, h_w^2(y) \qquad (2.4)$$

gives the Planck mass, related to the Newton constant by $8\pi G = M_P^{-2}$.

To obtain the action for the zero mode of $H_{\mu\nu\lambda}$ we note that (since there is only one harmonic zero mode in the internal space, i.e. the constant function 1):

$$H \wedge \star H = h_w^{-6} H \wedge \bar{\star} H, \qquad (2.5)$$

where $\bar{\star}$ is constructed from $\bar{g}_{\mu\nu}$, independent of the warp factor.

Correspondingly, the action for the zero mode of the NS-NS field $H = H(x^\alpha)$ is given by

$$S_{NS}^{(0)} = -\frac{1}{4\kappa^2} \int H \wedge \bar{\star} H \int d^6y \, \sqrt{g_{(6)}} \, e^{-\phi} h_w^{-2}(y) \,. \qquad (2.6)$$

Comparing (2.4) and (2.6), we see that in a flat background where $h_w = e^{-\phi} = 1$, the zero modes of both the graviton and the NS-NS three-form are Planck suppressed. This is to be expected, since both of them belong to the massless sector of the closed string theory in ten dimensions. However, as observed in [19–21], they appear with different normalizations in a warped background. In order to take this difference into account, we define the parameter β such that [21]

$$\beta = \frac{\int d^6y \, \sqrt{g_{(6)}} \, e^{-\phi} h_w^{-2}(y)}{\int d^6y \, \sqrt{g_{(6)}} \, h_w^2(y)}, \qquad (2.7)$$

where the action for the zero mode of the NS-NS field is now given by

$$S_{NS}^{(0)} = -\frac{\beta \, M_P^2}{4} \int H \wedge \bar{\star} H. \qquad (2.8)$$

The Bianchi identity for the gauge-invariant field H is

$$dH = \frac{\alpha'}{4} \left(\text{tr} \, R \wedge R - \frac{1}{30} \text{Tr} \, F \wedge F \right). \qquad (2.9)$$

To incorporate the axion in our construction, we dualize the B-field in the four-dimensional action by a scalar field a, via the following Lagrange multiplier for the Bianchi identity:

$$\int a \left[dH - \frac{\alpha'}{4} \left(\text{tr} \, R \wedge R - \frac{1}{30} \text{Tr} \, F \wedge F \right) \right]. \qquad (2.10)$$

The action containing the B-field kinetic energy and the Lagrange multiplier is

$$S = -\frac{\beta M_P^2}{4} \int H \wedge \bar{\star} H + \int a \left[dH - \frac{\alpha'}{4} \left(\text{tr} R \wedge R - \frac{1}{30} \text{Tr} F \wedge F \right) \right]. \qquad (2.11)$$

Integrating out H in terms of the field a one obtains

$$\bar{\star} H = \frac{2}{\beta M_P^2} \, da \,. \qquad (2.12)$$

This is equivalent to the statement that in four dimensions the axion is Hodge dual to the anti-symmetric $B_{\mu\nu}$ field. Plugging this into the action (2.11) yields

$$S(a) = \frac{2}{\beta M_P^2} \int d^4x \left(-\frac{1}{2} \partial_\mu a \partial^\mu a \right) + \int a \frac{\alpha'}{4} \left(\frac{1}{30} \text{Tr } F \wedge F - \text{tr } R \wedge R \right). \quad (2.13)$$

Upon rescaling the axion as in (1.3) and noting that $2\pi\sqrt{\alpha'} = m_s^{-1}$, we find

$$f_a = \sqrt{\frac{2}{\beta}} \frac{m_s^2}{M_P}. \quad (2.14)$$

In an unwarped compactification with $\beta = 1$ and taking $m_s/M_P \simeq 1/18$ in order to get the right GUT scale from string theory, one obtains $f_a \simeq 10^{16}$ GeV as in [13], too big to be acceptable. However, β can be significantly greater than one in a warped compactification. From (2.14) we see that this can reduce f_a to the range $10^9 - 10^{12}$ GeV. In subsequent sections we will provide specific warped examples where β is calculated to be large enough such that f_a falls within the desired window.

2.2 Example: heterotic compactification on a non-Kähler manifold

There have been many attempts to embed an axion construction in string theory (for an extensive review see [13] and references therein). Starting with a string scale comparable to M_P, it can be shown [13] that in most cases f_a is close to the GUT scale. The exceptions are when the Standard Model gauge fields are supported on vanishing cycles, where it is possible to lower f_a to the desired phenomenological bound [13].

Axion construction in very large compactification volumes was studied in [12]. A very large compactification corresponds to a low-scale string theory. It is argued that up to numerical factors of order unity, $f_a \sim m_s \sim 10^{11}$ GeV.

The axion in a warped heterotic background was considered in [14]. The model considered there is a heterotic compactification on a non-Kähler manifold [22–28]. The non-Kähler background is a non-trivial T^2 fibration over a K3 base. In the Einstein frame the full ten-dimensional metric can be written in the following way:

$$ds^2 = e^{-\frac{\phi}{2}} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-\frac{\phi}{2}} \left[(dx + \alpha_1)^2 + (dy + \alpha_2)^2 \right] + e^{\frac{3\phi}{2}} ds_{\text{K3}}^2, \quad (2.15)$$

where x and y are local coordinates such that $dx + idy$ is a holomorphic form on the T^2 fibers, and the α_i are local one-forms on the K3 base. For this particular compactification we see that the dilaton is related to the warp factor via $e^{-\phi} = h_w^4$. Plugging this into our expression for β in (2.7), one finds that $\beta = 1$. As mentioned in [14], the warping does not help to reduce f_a for the model-independent axion in the above background. This indicates that the axion decay constant is large if one starts with a large string scale. One way out of this conclusion is to construct a background where the dilaton is independent of the warp factor, so β can be made sufficiently large. The backgrounds studied in sections 3 and 4 of the present work both satisfy this condition and we bypass the difficulty of cancellation of the warp factor in (2.7) mentioned above.

2.3 Example: "model-dependent" axions in IIB

In this work we focus on the construction of a "model-independent" axion. However, one may naturally ask whether "model-dependent" axions with the allowed decay constants can be constructed in our formalism. The answer seems to be affirmative,[1] although in this paper we will only address this issue briefly because it requires us to know the details of the cohomological and homological properties of the internal space.

To give one example, consider a model where the axion arises from the zero mode of the RR four-form potential $C_{(4)}$ in type IIB string theory, say for example the background [29]. To support the axion we need a D7-brane such that

$$\int C_{(4)} \wedge F \wedge F \tag{2.16}$$

is non-zero, where the integration is over the D7 worldvolume. It is assumed that F has legs along the Minkowski coordinates, while $C_{(4)}(x^\alpha, y^m)$ has legs entirely along the compactified directions with x^α and y^m denote the coordinates on the Minkowski and the internal spaces respectively. This means that we are decomposing C_4 as

$$C_4(x^\alpha, y^m) \; = \; \varphi(x^\alpha) \otimes h_4(y^m) \tag{2.17}$$

where $h_4(y^m)$ is a harmonic four-form in the internal space and $\varphi(x^\alpha)$ is a scalar which will have axion-like couplings. Clearly, since the harmonic four-forms in the internal space are classified by the second Betti numbers b_2, there are b_2 axions from this decomposition. In the following we will choose $b_2 = 1$ to get a single axion for our case, but this is of course a model-dependent statement. Furthermore, it is also easy to see that in terms of powers of the warp factor h_w, the kinetic energy of the axion scales like

$$\int \sqrt{g_{(6)}} \, h_w^2 |h_4|^2 \, . \tag{2.18}$$

where $|h_4|$ is the magnitude of the harmonic four-form in the internal space. Using Hodge duality, this could be mapped to the harmonic $(1, 1)$ form, h_2, in the internal space. Now, constructing β using (2.7) we see that:

$$\beta = \frac{\int d^6y \, \sqrt{g_{(6)}} \, h_w^2(y) |h_2|^2}{\int d^6y \, \sqrt{g_{(6)}} \, h_w^2(y)}, \tag{2.19}$$

which may be significantly bigger than one depending on the behavior of the harmonic two-form in the internal space. Unfortunately, the exact form of h_2 for a CY (or non-CY) manifold is not known so we cannot make any concrete statement here.[2] Secondly, we require the harmonic form to be peaked near the throat of our internal space, which again requires us to know the precise form for the h_2. We do believe however that there is no conceptual problem in finding some internal space that can give rise to the required h_2 forms.

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[1]We thank Joe Conlon for discussion on this issue.

[2]To determine the harmonic form we need the metric of the internal space (say CY). So far there is no known solution for the metrics of compact CY spaces.

This conclusion can also be generalized to the zero modes of other "model-dependent" axions. The upshot is that model-dependent axions look like scalars from a four-dimensonal point of view and, as one can easily check, β for a scalar is dependent on the warp factors as well as the magnitude of the harmonic forms in the internal space. Tuning these correctly we can possibly have a large β from such compactifications although the results depends crucially on our knowledge of these harmonic forms (which is lacking at this stage).

The model-independent axion on the other hand is an exception: we are not required to know the detailed topological properties of the internal space as there is one and only one harmonic zero form in the internal space (which can be set to 1). Additionally, as mentioned in [19, 20], tensors of rank three and higher have the interesting property that their zero modes are strongly suppressed compared to the graviton zero mode.

Our goal in the rest of this paper is to construct warped geometries in heterotic string theory to see whether β for the model-independent axion $B_{\mu\nu}$ can be made sufficiently large to lower f_a into the narrow phenomenological window.

3. An AdS-type background in heterotic theory

As explained above, in order to get large enough β we need to construct warped geometries where the dilaton is independent of the warp factor. We construct backgrounds with precise warp factors using certain identifications between (p, q) and $(0, q)$ sigma models where $p = q = 1, 2, 4$ for bulk $\mathcal{N} = 1, 2, 4$ supersymmetries respectively. A class of the resulting heterotic backgrounds will resemble warped AdS$_5$ backgrounds. In the following we will discuss the sigma model identifications that we shall use to construct our backgrounds. Readers interested only in the final result should skip section 3.1 and go directly to section 3.2 where our backgrounds are presented. In section 4 we present another new heterotic background which has a warp-independent dilaton but non-trivial torsion.

3.1 Sigma-model constructions

Our present analysis will require us to study non-linear sigma models in both type II and heterotic theories, so we begin with a brief review of the method. First note that there are two ways to drag a type IIB background to the heterotic side:

- U-dualise a type IIB toroidal orientifold background to heterotic, or

- use sigma model identification to bring a torsional type IIB background to heterotic.

The two techniques achieve similar goals, but each comes with distinct advantages and disadvantages. To begin with, the first technique demands a ten-dimensional framework while the second works only from the two-dimensional point of view. Secondly, both techniques will in general involve some kind of U-duality needed to bring the type IIB theory into the required form. For the first, one must find an orientifold limit of a given type IIB background. This is not always easy. Being able to lift a given type IIB background to F-theory in principle guarantees the existence of an orientifold point but the lifting is not always easy to realise in practice. Orientifolding in the presence of background fluxes

implies projecting out some components of the fluxes, and therefore finding a consistent orientifold corner of a given F-theory compactification can turn out to be subtle.

The second technique might at first appear easier than the first, but this is not always the case since not every type IIB background can be pulled to the heterotic side. In particular, only specific choices of IIB fluxes are allowed by such an operation, so the presence of background fluxes can prove an obstacle. Since a generic warped type IIB compactification may therefore not have a heterotic dual in the latter, i.e. sigma model sense, it is sometimes necessary to modify the type IIB background in order to be able to find a heterotic dual. In general, a judicious choice of the appropriate technique should allow us to find the required heterotic dual from type IIB theory.

Our aim in the following analysis is to develop a heterotic background with say $(0, p)$ worldsheet supersymmetry from a given type IIB background with (p, p) worldsheet supersymmetry. The simplest way to do this is to add non-interacting fields to the sigma-model action with (p, p) supersymmetry. In general this will ruin the carefully balanced (p, p) supersymmetry of this model. We can use this to our advantage by adding non-interacting fields *only* in the left-moving sector. This breaks the left-moving supersymmetry, and one can therefore hope to obtain an action for a $(0, p)$ model from the (p, p) model.

To make this precise, let us consider the simplest case with $p = 2$ and define the corresponding sigma-model action in the following way:[3]

$$S = \frac{1}{8\pi\alpha'} \int d^2\sigma \left[(g_{ij} + B_{ij})\partial_+ X^i \partial_- X^j + \frac{1}{4} S^g_{\text{fermionic}} \right],$$ (3.1)

where the X^i are the bosons and $S^g_{\text{fermionic}}$ denotes the fermionic part of the action. Written this way, the action requires no other corrections[4] and will consequently be anomaly free. The fermionic part is made of a right-moving sector containing eight fermions (which we denote as ψ^p) and a left-moving sector also containing eight fermions (which we denote as $\psi^{\dot{q}}$). Together they give rise to the type II worldsheet action.

Taking into account the NS three-form fields H_{NS}, the fermionic part of a Green-Schwarz superstring can be written as

$$S^g_{\text{fermionic}} = 4i\psi^p \Delta_+ \psi^p + 4i\psi^{\dot{q}} \Delta_- \psi^{\dot{q}} + R_{(+)ijkl}\sigma^{ij}_{\dot{p}\dot{q}}\sigma^{kl}_{rs}\psi^{\dot{p}}\psi^{\dot{q}}\psi^r\psi^s,$$ (3.2)

where ψ^p and $\psi^{\dot{q}}$ are the two inequivalent spinor representations of the transverse D_4 and the sigma matrices are defined as $\sigma^{ij}_{\dot{p}\dot{q}} \equiv \Gamma^{[i}_{r[\dot{p}}\Gamma^{j]}_{\dot{q}]r}$ with a similar definition for the other components. The Gamma matrix has 8×8 blocks given as

$$\Gamma^i \equiv \begin{pmatrix} 0 & \Gamma^i_{p\dot{q}} \\ \Gamma^i_{\dot{r}s} & 0 \end{pmatrix},$$ (3.3)

which are used to define the σs above. In this notation we have to specify what we mean by the covariant derivative $\Delta_\pm$ and the curvature $R_{(+)ijkl}$. The most generic definition of

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[3]For $p = 1$ the situation is a little subtle as we will discuss later.
[4]For example Chern-Simons corrections.

the covariant derivative is given by [30, 24]

$$\Delta_\pm \psi^{q(\dot{q})} = \partial_\pm \psi^{q(\dot{q})} + \frac{1}{2}\left(\omega - \frac{1}{2}H\right)^{ab} \sigma_{ab}^{pq(\dot{p}\dot{q})}\psi^{p(\dot{p})}, \tag{3.4}$$

where $\frac{1}{2}H$ is the torsion and we have chosen the torsional connection $\omega_+ \equiv \omega - \frac{1}{2}H$, and not $\omega_- \equiv \omega - \frac{1}{2}H$ (see [24] for details on this). Observe that in the absence of H_{NS} there is no such ambiguity in the definition of the GS superstring. Finally, the curvature $R_{(+)ijkl}$ that we defined above is measured w.r.t. the connection ω_+, where

$$R^i_{(+)jkl} = \partial_k \Gamma^{(+)i}_{lj} + \Gamma^{(+)i}_{km}\Gamma^{(+)m}_{lj} - (k \leftrightarrow l), \tag{3.5}$$

and $\Gamma^{(+)i}_{jk}, H^i_{jk}$ are defined as

$$\Gamma^{(+)i}_{jk} \equiv \left\{{}^i_{jk}\right\} + \frac{1}{2}H^i_{jk}, \qquad H^i_{jk} = g^{il}H_{jkl}. \tag{3.6}$$

The above therefore gives the classical action for the type II string with background NS three-form fluxes. To get the (0, 2) heterotic action from the type II action described above, we take the following steps:

- Keep the right-moving sector unchanged, i.e. the ψ^p remain as before.

- In the left-moving sector, replace the $\psi^{\dot{q}}$ by eight fermions Ψ^A, $A = 1, \dots 8$. Also add 24 additional non-interacting fermions Ψ^B, $B = 9, \dots 32$.

- Replace ω_+ by the gauge field A, i.e. embed the torsional spin connection into the gauge connection.

The above set of transformations will convert the classical type II action given in (3.1) to a classical heterotic one i.e. (0, 2) one. One might, however, wonder about the Bianchi identity in the heterotic theory. The type IIB three-form fields are closed, whereas heterotic three-form fields satisfy the Bianchi identity. These statements seem to be reconciled by the embedding $\omega_+ = A$, which should result in a closed heterotic three-form. However, because of subtleties discussed in [24, 31, 32], the above embedding will not allow any compact non-Kähler manifolds in the heterotic theory, so this embedding is not an admissible solution to the problem. Therefore, as a first approximation, we assume an embedding of the form

$$\omega_+ = A + \mathcal{O}(\alpha'). \tag{3.7}$$

Using this, the new action with (0, 2) supersymmetry becomes

$$S = \frac{1}{8\pi\alpha'}\int d^2\sigma \bigg[(g_{ij} + B_{ij})\partial_+ X^i \partial_- X^j + i\psi^p(\Delta_+\psi)^p + i\Psi^A(\Delta_-\Psi)^A +$$
$$+ \frac{1}{2}F_{ij(AB)}\sigma^{ij}_{pq}\,\psi^p\psi^q\Psi^A\Psi^B + \mathcal{O}(\alpha')\bigg], \tag{3.8}$$

where due to the embedding (3.7), F^a_{ij} is the Yang-Mills field strength measured w.r.t. Lie algebra matrices T^a_{AB}. The fermion indices are $A = 1, \dots, 32$, which means there are 32

fermions, and hence the T^a form tensors of rank 16. The reader can easily identify the above action as an action for a heterotic sigma model with torsion [24, 26, 30, 33–35]. The action of the Laplacian (3.4) changes accordingly to

$$\Delta_- \Psi^A = \partial_- \Psi^A + A_i^{AB}(\partial_- X^i) \, \Psi^B;$$
$$\Delta_+ \psi^p = \partial_+ \psi^p + \frac{1}{2}(\omega_+)^{ab}\sigma_{ab}^{pq}\psi^q; \tag{3.9}$$
$$H_{ijk} = \frac{1}{2}\left(B_{ij,k} + B_{jk,i} + B_{kj,i}\right).$$

At this stage the replacements (3.9) may lead us to think that the sigma model has only (0, 1) supersymmetry. This is only superficial, and the situation is similar to the (1, 1) action for the type II case [30]. The full (0, 2) susy will be determined by additional actions on the fields (exactly as for the (1, 1) case). In the absence of torsion the above action (3.8) takes the following familar form:

$$S = \frac{1}{8\pi\alpha'} \int d^2\sigma \left[\delta_{ij}\partial_+ X^i \partial_- X^j + i\psi^p \partial_+ \psi^p + i\Psi^A \partial_- \Psi^A \right], \tag{3.10}$$

with the equations of motion

$$\partial_+ \psi^p = 0; \qquad \partial_- \Psi^A = 0, \tag{3.11}$$

that give us the orientations of the two worldsheet fermions.

Classically the action (3.8) is invariant under the following transformations:

$$\delta\Psi^A = \Lambda^A_B(X)\Psi^B, \qquad \delta A_i^{AB} = \partial_i \Lambda^{AB} + 2A_i^{C[A}\Lambda^{B]C},$$
$$\delta\Psi^P = \frac{1}{2}\Theta^{ab}(X)\sigma_{ab}^{pq}\Psi^q, \qquad \delta\omega_i^{ab} = \partial_i\Theta^{ab} + 2\omega_i^{c[a}\Theta^{b]c}, \tag{3.12}$$

where Λ denotes the local group rotations and Θ denotes local SO(8) Lorentz rotations. However, as is well known, these transformations are anomalous. The anomaly can be cancelled by the following choice of our three-form flux:

$$H = dB - \alpha'\left[\frac{1}{30}\text{Tr}\left(A \wedge F - \frac{1}{3}A \wedge A \wedge A\right) - \text{tr}\left(\omega_+ \wedge R_{(+)} - \frac{1}{3}\omega_+ \wedge \omega_+ \wedge \omega_+\right)\right], \tag{3.13}$$

which is of course the correct description for the torsional three-form H in the heterotic theory. Observe that the torsion H appears on *both* sides of the equation (3.13).

The above set of manipulations that convert a classical type II action to a classical heterotic one will help us to understand various things about the heterotic theory using data of type II theories. One important thing to notice is that both the metric and the B_{ij} fields can be taken to the heterotic side provided the original type II background does not have any H_{RR} fields. In the presence of H_{RR} the simple manipulations that we performed cannot give a (0, 2) or a (0, 1) model. We are therefore particularly interested in type II models that allow only for an NS three-form. When both NS and RR backgrounds are present, it might still be possible to perform the above manipulations if one can find an equivalent U-dual background. This U-dual background will in general not be Kähler

(not even conformally Kähler). Observe that these U-dualities do not require the original background to be at the orientifold point. This is therefore different from the analysis performed in [22, 23].[5]

Yet another issue is the existence of the bundle. This can be inferred from the detailed analysis given (at least for the U(1) case) in [26, 37]. However, the situation here may become a little simpler than the one in [26], because of the embedding (3.7). Recall that for the manifold studied in [26], a given complex structure J must satisfy the following constraint

$$i\partial\bar{\partial}J = 0 + \mathcal{O}(\alpha'), \tag{3.14}$$

which means that we can study the stability of bundles using the recent analysis of Li and Yau [38] and Fu and Yau [39]. For our case we will analyse the vector bundle directly from the sigma model by coupling the closed string worldsheets to open string worldsheets. We will touch on this issue later in the paper.

To construct an explicit heterotic background we have to make sure that the susy variations of the gravitino field χ_i and the gaugino field λ^A vanish in this background under an arbitrary susy parameter $\eta(X)$. This translates to the following set of well-known conditions:

$$\delta\chi_i \equiv \partial_i\eta + \frac{1}{2}\left(\omega_i^{ab} + \frac{1}{2}H_i^{ab}\right)\Gamma_{[a}\Gamma_{b]}\eta = 0,$$
$$\delta\lambda^A \equiv F_{ij}^A\Gamma^{[i}\Gamma^{j]} = 0. \tag{3.15}$$

Therefore to conclude this section we see that we can drag a given type IIB background to the heterotic side using our sigma-model identifications provided under U-dualities the IIB background has a non-trivial NS three-form and a metric with no RR forms. In the following section we will provide concrete constructions of a class of such backgrounds.

3.2 Construction of new heterotic backgrounds

From section 3.1 we know that we require a type IIB background with non-trivial metric and NS three-form. We also require a dilaton independent of the radial coordinate[6] so that β can be made reasonably large. One background that immediately comes to mind is type IIB theory on AdS_5 space. However, the minimally supersymmetric AdS_5 background, i.e. $AdS_5 \times T^{1,1}$, given by Klebanov-Witten [40] cannot be easily pulled to the heterotic side using our sigma-model identification. The non-trivial fibrations of the internal space $T^{1,1}$ create extra fluxes under U-duality which prohibit a heterotic dual for this background. We are therefore left with the other choices: $AdS_5 \times S^5$ and $AdS_5 \times \frac{S^5}{\mathbf{Z}_n}$ with[7] $n = 2, 3, 4, 6$.

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[5]Another interesting question would be to allow for both NS and RR backgrounds in the S-dual type I picture. This is a highly restrictive scenario as the allowed values of NS fluxes in the S-dual type I picture leave us with only two discrete choices [36].

[6]The dilaton will in general not be a constant in the heterotic theory because of the presence of torsion. The only exception is when the torsion is generated by Chern classes as we shall see in section 4.

[7]As is well known, the $n = 2$ case is the simplest, with the $\mathbf{Z}_2$ action considered in a specific way [40]. In general the typical $\mathbf{Z}_n$ action involves extra seven branes in the picture [41, 42]. We will discuss this later.

Using our analysis in the previous section we now claim that in the heterotic theory we will have a background of the form

$$ds^2 = e^\phi ds^2_{AdS_5} + ds^2_{X^5}, \tag{3.16}$$

that satisfies all the requirements laid down in the previous section. Here ϕ is the dilaton that depends only on the coordinates of the internal space X^5 and not on the radial coordinate r. This non-trivial dilaton will be supported by a background torsion H.

To be specific, our earlier arguments then require us to start with an $AdS_5 \times S^5$ background in type IIB string theory given (in units of α') by

$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2, \tag{3.17}$$

where $\mu, \nu = 0, 1, 2, 3$ are the spacetime directions and R is the curvature radius of the AdS space given by

$$R^4 = 4\pi g_s N. \tag{3.18}$$

Here N is the quantised charge of the five-form F_5:

$$\int_{S^5} F_5 = (4\pi^2\alpha')^2 N. \tag{3.19}$$

Furthermore, in the absence of NS and RR three-forms the five-form F_5 can be written as $F_5 = dC_{(4)} + *dC_{(4)}$ with the RR four-form, $C_{(4)}$, given by

$$C_{(4)} = \frac{r^4}{g_s R^4} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3. \tag{3.20}$$

Finally the metric of the five-sphere, $d\Omega_5^2$, in (3.17) is

$$d\Omega_5^2 = d\gamma^2 + \cos^2\gamma \, d\varphi_3^2 + \sin^2\gamma \left(d\psi^2 + \cos^2\psi \, d\varphi_1^2 + \sin^2\psi \, d\varphi_2^2\right), \tag{3.21}$$

where $0 \le \gamma, \psi \le \frac{\pi}{2}$ and $0 \le \phi_i \le 2\pi$. We see that there are three local isometries along directions φ_1, φ_2 and φ_3. We can choose φ_1 and φ_2 as the directions along which to perform our T-dualities, but we have to take care because there are no global one-cycles in the manifold. In fact, at the points

$$\gamma = 0; \quad \psi = 0; \quad \psi = \frac{\pi}{2}, \tag{3.22}$$

the cycles all shrink to zero size and the U-dual manifold will be non-compact. To avoid these issues, we will make our U-dualities away from the points (3.22). Now using the sigma model identifications, we find the following background in heterotic theory:

$$ds^2 = \frac{1}{2}R^2\sin^2\gamma \, \sin 2\psi \left[\frac{r^2}{R^2}dx_\mu dx^\mu + \frac{R^2}{r^2} dr^2\right]$$
$$+ \frac{1}{2}R^4\sin^2\gamma \, \sin 2\psi \left[d\gamma^2 + \cos^2\gamma d\varphi_3^2 + \sin^2\gamma \, d\psi^2\right] + \tan\psi \, d\varphi_1^2 + \cot\psi \, d\varphi_2^2;$$

$$e^\phi = \frac{1}{2g_s}\left(R^2 \sin^2\gamma \sin 2\psi\right);$$

$$H = \frac{4r^3}{R^4}e^{2\phi} \ * \left(dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dr \wedge d\varphi_1 \wedge d\varphi_2\right) + \mathcal{O}(\alpha')$$

$$= -\frac{4R^4}{g_s^2}\sin^3\gamma \cos\gamma \sin\psi \cos\psi \, d\gamma \wedge d\psi \wedge d\varphi_3 + \mathcal{O}(\alpha'),$$

(3.23)

with an additional vector bundle that we will describe later. This vector bundle has to satisfy the modified DUY equations which appear because of the background torsion [24, 26]. Furthermore our Hodge star operation is defined for a generic p-form in the following way:

$$(*\omega)_{\mu_1\mu_2......\mu_{10-p}} = \frac{\sqrt{-g}}{p!}\,\epsilon_{\mu_1\mu_2......\mu_{10-p}}{}^{\nu_1\nu_2......\nu_p}\omega_{\nu_1\nu_2......\nu_p}.$$

(3.24)

Observe that the new background on the heterotic side is not quite an AdS_5 background because of the unusual warp factors, although the radial dependence remains like that of the standard type IIB AdS_5 background. The internal space is also not an S^5 anymore. The metric has non-trivial warp factors that make the background non-Kähler. Furthermore, the dilaton is not a constant, and H is in general more complicated than the standard form[8]

$$H = e^{2\phi} * \left[d(e^{-2\phi}J)\right].$$

(3.25)

However we do expect the anomaly-cancelling Bianchi identity with $R_{(+)}$ and a vector bundle with curvature F defined on the internal six-dimensional space, to hold as:

$$dH = \frac{\alpha'}{4}\left(\text{tr } R_{(+)} \wedge R_{(+)} - \frac{1}{30}\text{Tr } F \wedge F\right).$$

(3.26)

To read off physical quantities, we transform the metric into the Einstein frame via $g_{MN}^{(E)} = e^{-\phi/2}g_{MN}^{(S)}$. After restoring the necessary factors of α' and rescaling x^μ ($g_s^{1/4}\alpha'^{1/2}\,x^\mu \to x^\mu$), (3.23) in the Einstein frame is given by

$$ds^2 = \sin\gamma\,\sqrt{\sin\psi\cos\psi}\left[\frac{r^2}{R}dx^\mu dx_\mu + \alpha'\sqrt{g_s}R^3\left(\frac{dr^2}{r^2} + d\gamma^2 + \cos^2\gamma d\phi_3^2 + \sin^2\gamma d\psi^2\right)\right]$$

$$+ \frac{\alpha'\sqrt{g_s}\sin^{\frac{1}{2}}\psi}{R\cos^{\frac{3}{2}}\psi\sin\gamma}d\phi_1^2 + \frac{\alpha'\sqrt{g_s}\cos^{\frac{1}{2}}\psi}{R\sin^{\frac{3}{2}}\psi\sin\gamma}d\phi_2^2,$$

(3.27)

with H given as in (3.23). Note that this geometry has the form of a warped metric (2.1) with warp factor

$$h_w^2 = \sin\gamma\sqrt{\sin\psi\cos\psi}\,\frac{r^2}{R}.$$

(3.28)

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[8]Recall that in the derivation of the dimensional reduction of a ten-dimensional heterotic action on a manifold with torsion [43] it is assumed that the warp factor is exactly equal to the dilaton (as in [33] for example). For us this is not the case, so there will be corrections. Additionally our background (3.23) does not have the standard form of G-structures discussed in [44]. We will address these issues elsewhere.

One can check that the background given by (3.27) or equivalently (3.23) is a consistent solution. With H given as in (3.23), the equation of motion $d \star H = 0$ is trivially satisfied. The dilaton equation,

$$\frac{1}{\sqrt{-g}} \partial_M \left(\sqrt{-g} \partial^M \phi \right) + \frac{e^{-\phi}}{12} H^2 = 0, \qquad (3.29)$$

is also satisfied. Finally the Einstein equation, $G_{MN} = \frac{1}{2} T_{MN}$, where G_{MN} is the Einstein tensor and T_{MN} is the stress-energy tensor given by

$$T_{MN} = \partial_M \phi \, \partial_N \phi - \frac{1}{2} g_{MN} \partial_P \phi \, \partial^P \phi + \frac{e^{-\phi}}{2} H_{MPQ} H_N^{\ PQ} - \frac{e^{-\phi}}{12} g_{MN} H^2, \qquad (3.30)$$

is also satisfied.[9] This gives us a powerful test of the consistency of our background. As mentioned above, the background (3.23) is well defined away from the points (3.22). Our sigma-model identification clearly fails at these points. It is then no surprise that the Ricci scalar for the metric (3.27), given by

$$\mathcal{R} = \frac{1 + 4 \sin^2 \psi \cos^2 \psi \sin^2 \gamma}{2\alpha' \sqrt{g_s} R^3 \sin^3 \gamma \sin^{5/2} \psi \cos^{5/2} \psi}, \qquad (3.31)$$

diverges at the points (3.22) mentioned above.

Such divergences can easily be cured by removing these points from the original S^5 (3.21). Then the metric (3.23) is a good description of the geometry away from these points, and the global six-dimensional manifold will be a compact non-Kähler manifold when we cut off the radial direction and replace it with a smooth cap. Physically, the quantity $g_s N$ corresponds to the integral of the three-form over a three-cycle of our non-Kähler manifold. As mentioned above, the four-dimensional spacetime is then a warped Minkowski spacetime with warp factor given by (3.28).

Once the singular points are smoothed out, the manifold will have a well-defined Riemann tensor globally. In fact there will now be an $\mathcal{O}(\alpha')$ correction to the torsion H (3.23) from the Riemann tensor two-form $\mathcal{R}_\omega$ that is linear in the torsion one-form. The total torsion $\mathcal{H}$ for the smooth manifold will then be a combination of the result (3.23) and the new contribution from the two-form $\mathcal{R}_\omega$. This is given by:

$$\mathcal{H} \ \equiv \ H + \frac{\alpha'}{2} \, \mathrm{Tr} \left(\tilde{H}_{(3)} \wedge \mathcal{R}_\omega + \dots \right) + \mathcal{O}(\alpha'^2), \qquad (3.32)$$

where H is defined in (3.23), $\tilde{H}_{(3)}$ is the one-form created from H using the vielbeins and $\mathcal{R}_\omega$ is defined, using the gravitational spin-connection at zeroth order in α', in the following way:

$$\mathcal{R}_\omega \ = \ d\omega + \frac{2}{3} \omega \wedge \omega. \qquad (3.33)$$

Thus the trace in (3.32) is the natural trace over the holonomy group of the internal manifold. The dotted terms involve higher orders in $\tilde{H}_{(3)}$ whose discussion we postpone for the

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[9]More details on how this solution solves the Einstein equations are given in the appendix.

next section. Here we want to point out that the total torsion (3.32) is well defined for our non-Kähler manifold as long as $\mathcal{R}_\omega$ is well defined. It is also not closed, and satisfies

$$d\mathcal{H} \;=\; \mathcal{O}(\alpha'). \tag{3.34}$$

This $\mathcal{O}(\alpha')$ can come from two potential sources: from the curvature correction discussed above, and from the embedding (3.7). We expect these contributions to combine nicely to give us the expected Bianchi identity.

3.2.1 The vector bundle

As emphasised before, the embedding (3.7) implies that the spin connection cannot be embedded in the gauge connection to get a full non-Kähler geometry. So the question is to determine the precise vector bundle on the heterotic side. Using our sigma-model identification, all we now need is to add open string worldsheets. This is subtle because generic open string worldsheets would ruin the conformal invariance in the original type IIB picture. Now, parametrising a unit radius S^5 in type IIB by

$$z \;=\; \cos\gamma\, e^{i\varphi_3}, \qquad v \;=\; \sin\gamma\,\cos\psi\, e^{i\varphi_1}, \qquad \tilde{v} \;=\; \sin\gamma\,\sin\psi\, e^{i\varphi_2}, \tag{3.35}$$

we see that the S^5 can be written as

$$d\Omega_5^2 = |dz|^2 + |dv|^2 + |d\tilde{v}|^2, \tag{3.36}$$

from which it is clear that vector bundles in the heterotic side should correspond to seven-brane configurations that are stretched along z and wrap a non-trivial two-cycle in $(v, \tilde{v})$ space. This configuration should have no global charge, and the distances between the seven branes should be parametrised by the (φ_1, φ_2) coordinates. It is possible to preserve conformal invariance in this set-up, so that the IIB metric remains AdS_5 and the global symmetries are G_i. Under U-duality this will then give us the vector bundles in the heterotic side.

With these constraints the background (3.23) will allow the corresponding G_i vector bundles respectively. The original SO(32) or $E_8 \times E_8$ gauge symmetry of the heterotic string would then be broken to any of these gauge symmetries by the corresponding Wilson lines.

On the other hand, if the seven branes wrap a four-cycle in $(v, \tilde{v})$ space instead of a two-cycle there is a simple way to get conformally invariant backgrounds with open strings: these are the constant coupling backgrounds of [45, 46], whose AdS limits were worked out in [41, 42]. In terms of the bulk picture this is an F-theory compactification with appropriate seven branes inserted [47]. Arranging these seven branes in certain special ways one can generate four distinct constant coupling backgrounds in type IIB theory (see [45, 46] for details). Out of the four allowed backgrounds associated with the global symmetries (D_4, E_6, E_7 and E_8), two — D_4 and E_7 — can be pulled to the heterotic side using our technique without generating extra fluxes. The final metrics for the two cases will look exactly the same as in (3.23) except that φ_3 will be periodic with period

$$0 \;\leq\; \varphi_3 \;\leq\; \pi \;\; \text{for} \;\; D_4 \quad \text{and} \quad 0 \;\leq\; \varphi_3 \;\leq\; \frac{\pi}{2} \;\; \text{for} \;\; E_7, \tag{3.37}$$

and the resulting picture will be a configuration of heterotic five-branes associated with these global symmetries. For the other two cases, namely global symmetries E_6 and E_8, the heterotic backgrounds are defined with

$$0 \leq \varphi_3 \leq \frac{2\pi}{3} \ \text{ for } E_6 \quad \text{and} \quad 0 \leq \varphi_3 \leq \frac{\pi}{3} \ \text{ for } E_8, \tag{3.38}$$

and with an additional two-form NS flux $B_{\mathrm{NS}} = \frac{1}{2}d\varphi_1 \wedge d\varphi_2$ with zero field strength. This flux is not globally defined because the φ_i are not globally defined coordinates. Furthermore the dilaton for these two cases differs slightly from (3.23) by an overall constant:

$$e^\phi = \frac{\sqrt{3}}{4}\left(R^2 \sin^2\gamma \sin 2\psi\right), \tag{3.39}$$

leading to another configuration of heterotic five branes.[10] Vector bundles in this set-up correspond to an intersecting configuration of seven branes, much like the one studied by Gimon and Polchinski [49]. A more detailed discussion of these issues will be addressed elsewhere.

3.2.2 Related AdS-type backgrounds

Before we end this section, we note that if we change the orientation of the three-form flux H from φ_1, φ_2 to φ_1, φ_3, keeping other factors unchanged, we can generate a slightly different background that falls in the same class as (3.23):

$$
\begin{aligned}
ds^2 &= \frac{1}{2}R^2 \sin 2\gamma \cos\psi \left[\frac{r^2}{R^2}dx_\mu dx^\mu + \frac{R^2}{r^2}\,dr^2\right] \\
&\quad + \frac{1}{2}R^4 \sin 2\gamma \cos\psi \left[d\gamma^2 + \sin^2\gamma\, d\psi^2 + \sin^2\gamma \sin^2\psi\, d\varphi_2^2\right] \\
&\quad + \tan\gamma \cos\psi\, d\varphi_3^2 + \cot\gamma \sec\psi\, d\varphi_1^2; \\
e^\phi &= \frac{1}{2g_s}\left(R^2 \sin 2\gamma \cos\psi\right); \\
H &= -\frac{4R^4}{g_s^2}\sin^3\gamma \cos\gamma \sin\psi \cos\psi\, d\gamma \wedge d\psi \wedge d\varphi_2 + \mathcal{O}(\alpha').
\end{aligned}
\tag{3.40}
$$

This metric also has singularities. The Ricci scalar for this background in the Einstein frame scales like $\mathcal{R} \sim \sin^{-5/2}\gamma \cos^{-5/2}\gamma \sin^{-5/2}\psi$, which diverges at $\gamma = 0$, $\gamma = \pi/2$ and $\psi = \pi/2$. Excising these points, and cutting off the radial direction to replace it with a finite cap we can have a smooth non-Kähler manifold that has well-defined curvature forms. The full torsion for the manifold to higher orders in α' can now be easily computed following our earlier analysis (3.32).

Yet another background, also falling in the same class, can be derived by changing the orientation of the three-form from φ_1, φ_2 to φ_2, φ_3. This corresponds to taking $\sin\psi \rightarrow$

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[10]It is interesting to note that the Argyres-Douglas points [48] would correspond to the following periodicities for φ_3: $\left[0, \frac{5\pi}{3}\right]$, $\left[0, \frac{3\pi}{2}\right]$, and $\left[0, \frac{4\pi}{3}\right]$, associated respectively with A_0, A_1 and A_2 global symmetries in type IIB theory. An analysis of the metric, dilaton and torsion can easily be done for these cases too but we will not dwell on them here as our emphasis is more on studying axions than new non-Kähler manifolds.

$\cos \psi$ in (3.40):

$$ds^2 = \frac{1}{2}R^2 \sin 2\gamma \sin \psi \left[\frac{r^2}{R^2}dx_\mu dx^\mu + \frac{R^2}{r^2} \, dr^2 \right]$$
$$+ \frac{1}{2}R^4 \sin 2\gamma \sin \psi \left[d\gamma^2 + \sin^2\gamma \, d\psi^2 + \sin^2\gamma \, \cos^2\psi \, d\varphi_1^2 \right]$$
$$+ \tan \gamma \sin \psi \, d\varphi_3^2 + \cot \gamma \, \mathrm{cosec} \, \psi \, d\varphi_2^2;$$
$$e^\phi = \frac{1}{2g_s} \left(R^2 \sin 2\gamma \sin \psi \right);$$
$$H = -\frac{4R^4}{g_s^2} \sin^3 \gamma \cos \gamma \sin \psi \cos \psi \, d\gamma \wedge d\psi \wedge d\varphi_1 + \mathcal{O}(\alpha').$$
$$(3.41)$$

This geometry also has singularities at $\gamma = 0$, $\gamma = \pi/2$ and $\psi = 0$, and following the same procedure as before we can have a smooth non-Kähler geometry with torsion and non-trivial vector bundles.[11]

3.3 The axion decay constant

Having constructed specific warped AdS heterotic backgrounds we can calculate the normalization constant β from (2.7) and find the axion decay constant f_a from (2.14). The AdS geometries presented in the previous section should be considered as local warped regions or throats, which are glued in the UV to the compactification bulk. The warped throat is glued to the bulk at $r = L$, where for consistency we impose $R \simeq L$, where $R = (4\pi g_s N)^{1/4}$ is the AdS curvature radius of the AdS geometry. The overall size of the bulk of the compactification, R_6, is assumed to be much bigger than the size of the throat, $R_6 \gg L$, such that the bulk contains most of the volume of the compactification. Furthermore, it is assumed that the bulk is not warped.

The AdS geometry is also subject to an IR cut off, when $r \to 0$. There is a conical singularity at $r = 0$ and we assume that the geometry near the tip of the cone or throat is modified such that this singularity is smoothed out as in the Klebanov-Strassler (KS) background [50]. The IR geometry is cut off at $r = r_0$ and the value of the warp factor h_0 at r_0 (after integrating the angular directions) is given by

$$h_0 = \frac{r_0}{\sqrt{R}}. \qquad (3.42)$$

As in the KS solution, there are corrections to h_0 due to IR modification of the throat. We expect them to be sub-leading and that they will not play a significant role in our discussion.

Noting that $4\pi \kappa_{10}^2 = m_s^{-8}$ and defining the volume of the bulk to be $v_6 = R_6^6/4\pi$, the

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[11]Note however that there is an apparent obstruction to pulling the constant coupling backgrounds of [45, 46] to the heterotic side using our sigma-model identifications.

four-dimensional gravitational coupling from (2.4) is

$$M_P^2 = 4\pi m_s^8 \left(\frac{R_6^6}{4\pi} + \frac{1}{2}\pi^3 R^4 \alpha'^3 g_s^{\frac{3}{2}} (1 - \sin^4 \gamma_1)(\sin^2 \psi_2 - \sin^2 \psi_1)(L^2 - r_0^2) \right)$$

$$\simeq m_s^8 R_6^6 \left(1 + 2\pi^4 g_s^{\frac{3}{2}} \frac{R^4 L^2 \alpha'^3}{R_6^6} \right)$$

$$\simeq m_s^8 R_6^6 \,. \tag{3.43}$$

Here γ_1 and ψ_i are the cut-off values for the angular variables γ and ψ at the singular points (3.22). In going from the first line to the second line above, it is assumed that $r_0 \ll L$, $\gamma_1 \to 0$, $\psi_1 \to 0$ and $\psi_2 \to \pi/2$. To obtain the final answer, as mentioned before, it is assumed that the bulk contains most of the volume of the compactification, corresponding to $R^4 L^2 \alpha'^3 \sim L^6 \alpha'^3 \ll R_6^2$.

Similarly, for β we obtain

$$\beta = \left(\frac{R_6^6}{4\pi} \right)^{-1} \left[e^{-\phi_B} \frac{R_6^6}{4\pi} + 4\pi^3 R^4 \alpha'^3 g_s^{\frac{5}{2}} \left| \ln \left(\frac{\tan \psi_2}{\tan \psi_1} \right) \ln(\sin \gamma_1) \right| \left(\frac{1}{r_0^2} - \frac{1}{L_0^2} \right) \right]$$

$$\simeq \frac{g_s^{\frac{5}{2}}}{4\pi^2} \frac{m_s^2}{M_P^2} \left| \ln \left(\frac{\tan \psi_2}{\tan \psi_1} \right) \ln(\sin \gamma_1) \right| \frac{R^4}{r_0^2}\,. \tag{3.44}$$

In the first line, it is assumed that in the bulk the dilaton field does not change significantly, and we denote its value in the bulk by ϕ_B. To go from the first line to the second line it is assumed that $r_0/R^2 \ll 1$, so the second term dominates over the first term. Physically, this means that the normalization of the zero mode of B_{NS} gets its largest contribution from the highly warped throat [19–21]. This should be contrasted with the normalization of the graviton zero mode, which is insensitive to the warp factor [51]. The calculation of the four-dimensional gravitational coupling in (3.43) reflects this.

Plugging this value of β into (2.14), we obtain the axion decay constant

$$f_a \simeq \frac{\sqrt{8}\,\pi}{g_s^{5/4} R^{3/2}} \left| \ln \left(\frac{\tan \psi_2}{\tan \psi_1} \right) \ln(\sin \gamma_1) \right|^{-1/2} (h_0\, m_s)\,. \tag{3.45}$$

The dependence on the cut-off angles γ_1 and $\psi_{1,2}$ is expected on physical grounds: the cut-off represents the deformation of the geometry near the singularities. These deformations will eventually show up in β and f_a, when integrals over the non-singular compactification are performed. However, the axion decay constant is very insensitive to the angular coordinate cut-off; it depends only logarithmically on γ_1 and $\psi_{1,2}$. As long as one is not exponentially fine-tuning the cut-off parameters to their singular values, the logarithmic expressions in f_a will be of $\mathcal{O}(1)$. On the other hand, $R = (4\pi g_s N)^{1/4}$. For parameters of physical interest, one can assume $g_s \sim 0.1$ and $1 \lesssim g_s N \lesssim 100$ such that $R \gtrsim 1$. Combining all these in f_a, we obtain the following expression for the axion decay constant:

$$f_a = c\, h_0 m_s\,, \tag{3.46}$$

where c is a constant of order $1 - 10$, depending on the geometry of the throat. Recall that $h_0\, m_s$ is nothing but the physical mass scale at the bottom of the throat. This indicates that

the axion decay constant is controlled by the physical scale of the throat and is insensitive to the details of the bulk.

To obtain f_a within the acceptable range, i.e. $10^9 \, \text{GeV} \lesssim f_a \lesssim 10^{12} \, \text{GeV}$, all one has to do is to construct a throat in the string theory compactification with the physical mass scale within this range. This is easily achieved in the light of recent progress in flux compactifications [22, 52].

The situation becomes more interesting in a multi-throat compactification. One immediate conclusion of the result above is that in the multi-throat compactification, the normalization of the axion field (or $B_{\mu\nu}$ field) zero mode is controlled by the longest throat in the compactification. To obtain axion decay of the right scale, one has to make sure that the physical mass scale of the longest throat is within the range $10^9 - 10^{12}$ GeV. In the case where the physical scales of the throats are comparable, our formulation for calculating β indicates that

$$ f_a = \left[\sum_i c_i^{-2} h_{0\,i}^{-2} \right]^{-1/2} m_s \tag{3.47} $$

where the sum is over all throats. Here $h_{0\,i}$ represents the warp factor at the bottom of the i-th throat and the c_i are constants of order unity or so depending on the construction of the corresponding throat. Thus, even in the situation where the physical mass scale for each throat is bigger than 10^{12} GeV, all throats contribute to the normalization of the axion zero mode such that the sum in (3.47) can bring f_a within the desired range.

4. Constant coupling background

The previous examples we have studied give rise to large β provided we impose a reasonable cut-off when compactifying the geometry. Two important aspects of our previous analysis were firstly that the dilaton remained independent of the radial coordinate r, even though the background had non-trivial torsion, and secondly, that the analysis of f_a was insensitive to the cut-off. This was not surprising since local cut-offs in the geometry should not affect many of the global features of a system. However, it would be nice to construct a background with torsion in the heterotic theory that is compact from the beginning, and allows a dilaton that is (at least) independent of the radial coordinate r.

In the following we will sketch a possible background, with torsion and a constant dilaton. We will argue that in this background β as defined in (2.7) can be made significantly greater than one.

A brief discussion of this issue appeared in [28], where new heterotic backgrounds found via M-theory were studied. Our analysis here will similarly use M-theory to arrive at the heterotic theory, and will therefore be slightly different from the analysis presented in section 3. As discussed in [28], we want to emphasise that the localized and non-localized G-fluxes in M-theory play an important role in determing the precise backreaction effects on the underlying geometry. The conclusion was that the heterotic manifold will always be non-Kähler when only non-localized M-theory fluxes are present. On the other hand, in the presence of only localized fluxes, the manifold may or may not be Kähler. For our

case, we will take the following ansatz for the metric of the heterotic theory in the Einstein frame:

$$ds^2 = e^{\mathcal{D}} g_{\mu\nu} dx^\mu dx^\nu + g_{mn} dx^m dx^n, \tag{4.1}$$

where $\mathcal{D}$ is the warp factor, $\mu, \nu = 0, \ldots, 3$ and m, n denote internal coordinates of the manifold that may or may not be Kähler to begin with. The generic formula for the non-Kählerity is given in terms of the torsion classes $\mathcal{W}_i$ in the following way:[12]

$$dJ = \frac{3i}{4} \left(\mathcal{W}_1 \bar{\Omega} - \bar{\mathcal{W}}_1 \Omega \right) + \mathcal{W}_3 + J \wedge \mathcal{W}_4, \tag{4.2}$$

where Ω is the (3,0)-form for the manifold. If we demand that the internal manifold be complex then $\mathcal{W}_1 = \mathcal{W}_2 = 0$. For our case we will also demand that $dB = 0$ and vanishing cosmological constant. This means that the torsion will only come from the Chern-Simons term and we have a four-dimensional Minkowski space (see [54] for a discussion of cosmological constant from the torsion classes). The general formula for non-Kählerity can then be found from (4.2) to take the following form:

$$dJ = \alpha' * [\Omega_3(\omega_+) - \Omega_3(A)] + a_1 \, e^{-2\phi} d\phi \wedge J + a_2 \, d\mathcal{D} \wedge J, \tag{4.3}$$

where a_1 and a_2 are constant coefficients whose values could be determined from (4.2), A is the one-form gauge bundle and ϕ is the heterotic dilaton. Observe that we have not identified the dilaton ϕ with the warp factor $\mathcal{D}$. This is therefore more generic than the corresponding formula presented in [30, 33, 31].

Thus, even in the absence of a background three-form, the manifold can become non-Kähler due to the presence of vector bundles, a non-trivial dilaton and the warp factor. Kählerity is restored only when

$$\omega_+ = A, \qquad \phi = \text{constant}, \qquad \mathcal{D} = 0, \tag{4.4}$$

which are precisely the conditions studied in [55]. Here we have derived the conditions by demanding a Kähler compactification from the generic equation for dJ.

Let us now imagine that we do not turn on any non-localized gauge fluxes and at the same time do not allow the standard embedding. Then naively we would expect to get a non-Kähler manifold with the non-Kählerity coming precisely from the difference $\Omega_3(\omega_+) - \Omega_3(A)$ (and the dilaton and warp factor) where

$$\Omega_3(A) = \frac{1}{30} \text{Tr} \left(A \wedge F - \frac{1}{3} A \wedge A \wedge A \right), \tag{4.5}$$

and $\Omega_3(\omega_+)$ is defined equivalently but with a trace in the fundamental representation of the holonomy group. In fact, the three-form fluxes will typically look like [27, 26]

$$H_{ABC} = f_{ABC} + \frac{\alpha'}{2} \text{Tr} \left(\omega \wedge \tilde{f} \wedge \tilde{f} + \tilde{f} \wedge \mathcal{R}_\omega + \frac{1}{2} \tilde{f} \wedge d\tilde{f} - \frac{1}{6} \tilde{f} \wedge \tilde{f} \wedge \tilde{f} \right)_{ABC} + \mathcal{O}(\alpha'^2), \tag{4.6}$$

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[12]See [37, 53] for details about torsion classes.

where $f_{ABC} = \alpha'[\Omega_3(\omega) - \Omega_3(A)]_{ABC}$ and ω is the gravitational spin connection at zeroth order in α' (see [26] for more details). We have also defined $\tilde{f}$ as a one-form created from f using the vielbeins i.e $\tilde{f}_A{}^{ab} \equiv f_{ABC}e^{Ba}e^{Cb}$ where e^{Aa} is the vielbein, and $\mathcal{R}_\omega$ is defined in (3.33). In particular the traces are defined in the following way:

$$\mathrm{Tr}\left(\omega \wedge \tilde{f} \wedge \tilde{f}\right)_{ABC} \equiv \omega_{[A}{}^{ab}\, \tilde{f}_B{}^{cd}\, \tilde{f}_{C]}{}^{ef}\, \mathrm{Tr}\left(M_{ab}M_{cd}M_{ef}\right), \qquad (4.7)$$

where M_{ab} is the matrix representation of the holonomy group. Similar definitions work for the other terms in H_{ABC}.

The above three-form backreacts on the geometry to make the space non-Kähler. Thus, vector bundles without the standard embedding seem to be allowed only on non-Kähler manifolds, unless there is an additional α' correction that could cancel the Chern-Simons term in (4.3). At this point it would be interesting to compare the result with a similar construction discussed in [56] where a fractional gauge Chern-Simons term was switched on the internal manifold. The key additional ingredient was a background gaugino condensate.[13]

To conclude, the manifold can still become non-Kähler in the absence of flux via the relation (4.3). In [56] the $\Omega_3(\omega)$ term was cancelled by one of the Chern-Simons terms of the gauge fields. Therefore the non-Kählerity in this model arose from vector bundles of one of the E_8 gauge groups. To obtain a Calabi-Yau space we need $dJ = 0$. For this to be possible we would need to cancel the Chern-Simons term with the condensate. Fortunately the condensate is of the same order in α' as the Chern-Simons term. Furthermore, the presence of the condensate provides an additional contribution to the superpotential of [27, 43]. This additional contribution was worked out in [57, 27] following the work of [58] (see also [59] for some recent works). Additionally, the ten-dimensional Lagrangian for the heterotic theory will change from (2.2) to the following:

$$S = \frac{1}{2\kappa^2}\int d^{10}x\,\sqrt{-g}\Bigg[R - \frac{1}{2}\partial_M\phi\partial^M\phi - \frac{1}{12}e^{-\phi}\left(H_{ABC} - \frac{\alpha'}{16}e^{\frac{\phi}{2}}\bar{\chi}\Gamma_{ABC}\chi\right)^2 \quad (4.8)$$
$$- \frac{\alpha'}{120}e^{-\frac{\phi}{2}}\,\mathrm{Tr}\left(F_{AB}F^{AB}\right) - \frac{\alpha'}{30}\mathrm{Tr}\left(\bar{\chi}\Gamma^A D_A\chi\right)\Bigg],$$

where the χ^A are the gaugino fields of the heterotic theory. From here one can see that the equation for non-Kählerity is now given by

$$dJ = \alpha' * [\Omega_3(\omega_+) - \Omega_3(A)] + a_1\, e^{-2\phi}d\phi \wedge J + a_2\, d\mathcal{D} \wedge J + a_3\, \alpha' * \langle\bar{\chi}^A\Gamma\chi^A\rangle. \quad (4.9)$$

The above equation allows for the following scenarios:
• Cancelling the Chern-Simons term with the gaugino condensate[14] and the dilaton term with the warp factor would lead to a Kähler manifold. With the underlying SU(3) structure this would eventually become a Calabi-Yau manifold.

---

[13]The gaugino condensate contributes to the (3,0) and the (0,3) parts of the three-form, and can break susy, but we will consider a condensate that preserves susy.

[14]Of course the Chern-Simons term has to be of the form (3,0) + (0,3) with no (2,1) + (1,2) part. This is highly dependent on the complex structure, and one example of this is provided in [56].

- The cancellation of Chern-Simons term with the condensate is not always possible. However the dilaton and warp factor can be both made proportional to a constant. Then the manifold is Kähler to the zeroth order in α' and becomes non-Kähler to the first order in α' i.e. $dJ = \mathcal{O}(\alpha')$. It would be interesting to see whether the construction of [56] falls in this category or the previous one.

- The Chern-Simons term is cancelled by the condensate, but the dilaton is a constant. Then the manifold is non-Kähler due to the warp factor i.e. $dJ = \mathcal{O}(1)$. When the Chern-Simons term is not cancelled but the dilaton is still a constant then the manifold remains non-Kähler i.e. $dJ = \mathcal{O}(1) + \mathcal{O}(\alpha')$.

- Taking the contribution of the background three-form (4.6), one might be able to retain Kählerity to zeroth and first order in α'. This would mean that the manifold could become non-Kähler at second order in α' i.e. $dJ = \mathcal{O}(\alpha'^2)$. Of course non-Kählerity to zeroth and first order in α' is still possible.

In this paper we will only discuss the implication of the third case, i.e. heterotic compactification with a constant dilaton, non-trivial warp factor and a torsion generated by the Chern-Simons term only (a gaugino condensate would make the system more involved). The background is therefore

$$
\begin{aligned}
ds^2 &= e^{\mathcal{D}}dx_\mu dx^\mu + g_{mn}dx^m dx^n; \\
e^\phi &= \text{constant}; \\
H &= \frac{\alpha'}{120}\,\text{Tr}\,\left(\frac{1}{3}A_1 \wedge A_1 \wedge A_1 - A_1 \wedge F_1\right),
\end{aligned}
\tag{4.10}
$$

where A_1 is the gauge field of one E_8 and the spin connection ω_+ is embedded in the gauge connection A_2 of another E_8, i.e. $\omega_+ = A_2$. These manifolds are called *balanced* manifolds.[15]

The axion decay constant for this background is formally given by (2.14) where β is calculated from (2.7) with the replacement $h_w^2 \to e^{\mathcal{D}}$. To find f_a explicitly, we would need to know $e^{\mathcal{D}}$ and g_{mn} explicitly. However, motivated by our AdS_5 example, we believe that f_a for this background can also be lowered to within the narrow phenomenological bound. As explained before, the key property here is that we are able to construct a background, at least in principle, where the dilaton is independent of the warp factor. It would be interesting to work out the details of the supergravity equations for this background and verify explicitly that f_a can fall to within the desired window.

5. Conclusion

In the present work, the viability of axion constructions in warped heterotic backgrounds was studied. Manifolds considered in this context previously have in general been found to result in axion decay constants outside the allowed range, motivating the construction

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[15]We thank Evgeny Buchbinder for pointing this out to us.

and investigation of new warped AdS-type heterotic backgrounds and their effect on the axion decay constant. As far as we know, these backgrounds, of the form (in string frame)

$$ds^2 = e^\phi ds^2_{AdS_5} + ds^2_{X^5}, \qquad (5.1)$$

with non-trivial dilaton and torsion, have not been studied before. From the four-dimensional point of view these backgrounds are warped Minkowski spacetimes with warp factor given by (3.28). The compact six-dimensional manifolds are new non-Kähler manifolds, different from those studied in [22 – 26]. This work can therefore be viewed in two ways: as a study of the effect of warping in string-theory backgrounds meeting certain criteria on the axion decay constant or as the phenomenologically motivated construction of new compact non-Kähler backgrounds in heterotic string theory.

We gave one possible class of constructions of these backgrounds, which we believe to have smooth global metrics, using sigma-model identifications. It is interesting to note that the corresponding torsions can be expressed in powers of α' and satisfy the Bianchi identity in terms of the vector bundles and the torsional spin connections. We gave a simple way to construct vector bundles on these manifolds. A crucial feature of these backgrounds is that the dilaton is independent of the radial direction and therefore not quite proportional to the warp factor as in [22, 23, 33]. A second class of non-Kähler manifolds with similar properties was also presented, with the important difference that the torsion comes only from the Chern classes of the manifolds. These manifolds have been classified in the literature as either "conformally balanced" or "balanced" manifolds. Our case was an example of a balanced manifold with a constant dilaton but non-trivial torsion.

We argued that the zero modes of the "model-dependent" axions may also give values of f_a in the desired range, provided we have the precise knowledge of the harmonic two-forms i.e. h_2 in the internal space. These harmonic forms should be peaked near the throat of our internal space. Unfortunately, so far we have no precise knowledge of these forms and therefore we cannot make any definative statement here. We believe that one may be able to find the required internal manifolds with harmonic forms satisfying the required properties.

The axion in our construction is instead a "model-independent" axion, constructed from the zero mode of the NS-NS two-form potential $B_{\mu\nu}$. We have shown that the normalization of the zero mode of the $B_{\mu\nu}$ field gets most of its contribution from the highly warped regions of the compactification. It is on this fact that our mechanism for lowering the axion decay constant f_a to the desired range $(10^9 - 10^{12}\,\mathrm{GeV})$ hangs. More specifically, we have shown that f_a is given by the warped mass scale of the longest throat in the compactification. Hence, the question of achieving an axion construction with a viable value of f_a in this set-up is reduced to the question of constructing an AdS-type throat with mass scale in the desired $10^9 - 10^{12}\,\mathrm{GeV}$ range. As we explicitly demonstrated, heterotic compactification on any of the presented manifolds with appropriate torsion, dilaton and vector bundles should thus allow axions with permissible decay constants in a natural way.

Finally, one subtle issue that we have not addressed is the location of the standard model in our scenario. As is well known, there are two distinct ways we can argue for the generation of the standard model spectrum. The first is the well known dimensional

reduction of the SO(32) or $E_8 \times E_8$ gauge fields on the non-Kähler manifold. Using the metric ansatze (2.1) we see that the kinetic term of the gauge fields is *not* supressed by the warp factor h_w. However the interaction of the model-independent axion with the gauge fields is given by

$$\frac{\alpha'}{4\kappa^2} \int e^{-\phi}\, dB \wedge *\Omega_3(A),$$

(5.2)

where $\Omega_3(A)$ is the Chern-Simons term for the gauge field, and this is suppressed exactly as in (2.6). Thus the coupling of the axion (from the B field) and the standard model gauge fields (from $\Omega_3(A)$) is suppressed by the warp factor, and therefore the coupling scale is set by the scale of the throat.

The second way of generating the standard model, which is rather unconventional, is to use heterotic NS5 branes wrapped on two-cycles of the internal space. In the SO(32) heterotic theory the NS5 brane is the small instanton configuration and therefore has a direct gauge-theory spectrum on its world volume [60]. On the other hand for the $E_8 \times E_8$ heterotic theory the NS5 brane spectrum is a tensor multiplet [61]. Multiple NS5 branes could give us a non-abelian four dimensional gauge theory. Clearly such a theory could be localised at the throat of the manifold by allowing only suitably localised harmonic forms in the internal manifold. The axion coupling would then also be given by the scale of the throat instead of the bulk. Details on this will be presented elsewhere.

Acknowledgments

A. The Einstein equation

Here we give Einstein's equations for our solution in more detail. The Einstein equation is $G_{MN} = \frac{1}{2} T_{MN}$ where the stress energy tensor is given by (3.30):

$$T_{MN} = \partial_M \phi\, \partial_N \phi - \frac{1}{2} g_{MN} \partial_P \phi\, \partial^P \phi + \frac{e^{-\phi}}{2} H_{MPQ} H_N^{\ PQ} - \frac{e^{-\phi}}{12} g_{MN} H^2.$$

To be specific, we calculate the components of Einstein tensor for the background (3.27):

$$
\begin{aligned}
G_{00} &= -G_{ii} = \frac{r^2 \left(1 + 12 \sin^2 \psi \cos^2 \psi \sin^2 \gamma\right)}{4\sqrt{g_s}\alpha' R^2 \left(\sin^2 \gamma \cos^2 \psi \sin^2 \psi\right)}; \\
G_{rr} &= -\frac{R^4}{r^4} \sqrt{g_s}\alpha' G_{00}; \\
G_{\gamma\gamma} &= \frac{-1 - 12 \sin^2 \psi \cos^2 \psi \cos^2 \gamma + 20 \sin^2 \psi \cos^2 \psi}{4 \sin^2 \gamma \cos^2 \psi \sin^2 \psi}; \\
G_{\psi\psi} &= (8 - G_{\gamma\gamma}) \sin^2 \gamma; \\
G_{\phi 1 \phi 1} &= \frac{\sqrt{g_s}\alpha' G_{00}}{r^2 \sin^2 \gamma \cos^2 \psi};
\end{aligned}
$$

(A.1)

$$G_{\phi2\phi2} = \frac{\sqrt{g_s}\,\alpha' G_{00}}{r^2 \sin^2\gamma \sin^2\psi};$$

$$G_{\phi3\phi3} = \frac{\cos^2\gamma \left(-1 + 20\sin^2\psi \cos^2\psi \sin^2\gamma\right)}{4\sin^2\gamma \cos^2\psi \sin^2\psi};$$

$$G_{\gamma\psi} = \frac{\cos\gamma(\cos^2\psi - \sin^2\psi)}{\sin\gamma \sin\psi \cos\psi}.$$

One can check that the off-diagonal component of the Einstein tensor is sourced by $\partial_\psi\phi\partial_\gamma\phi$. With these values for G_{MN} along with H and ϕ given as in (3.23), one can explicitly check that the Einstein equations are all satisfied. This demonstrates that the background (3.27) is a genuine solution. Similarly, one can also check that backgrounds (3.40) and (3.41), when written in the Einstein frame, are also consistent solutions with the given forms of ϕ and H.

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